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Conversations in a professional learning community: An analysis of teacher learning opportunities in mathematics

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The growing perception of professional learning communities as an effective professional development approach needs to be supported with knowledge of how such communities create learning opportunities for teachers. Activities in professional learning communities are underpinned by collegial conversations that foster learning, and in this article we analysed such conversations for learning opportunities in one professional learning community of mathematics teachers. Data consisted of audio-recorded community conversations. The focus of the conversations was to understand the thinking behind learners' errors, and teachers engaged in a number of activities related to learner errors and learner reasoning. Our analyses show how opportunities for learning were created in identifying the origins of learners' errors as well as learners' thinking underlying their errors. Results also showed that the teachers had opportunities for learning how to identify learners' learning needs and in turn the teachers' own learning needs. The teachers also had opportunities for deepening their own understanding of the conceptual meaning of ratio. The learning opportunities were supported by the following: having a learning focus, patterns of engagement that were characterised by facilitator questioning, teacher responses and explanations, and sharing knowledge. Such mutual engagement practices in professional learning communities resulted in new and shared meanings about teachers' classroom practices. Our findings also show the critical role of a facilitator for teacher learning in professional learning communities.

Introduction

Teacher learning in professional learning communities is generally accepted as a teacher professional development approach that can significantly impact teachers' mathematical knowledge and practices (e.g. Brodie & Borko, 2016b; Horn, 2010; Koellner & Jacobs, 2015). In South Africa the move towards professional learning communities as enunciated in the Integrated Strategic Planning Framework for Teacher Education and Development (Departments of Basic Education and Higher Education and Training, 2011) marked a shift in teacher professional development policy, necessitated by the realisation that standalone workshops were not producing results in terms of improving teachers' knowledge, practices and learners' performance (Bertram, 2011; McCarthy & Oliphant, 2013; Nel, 2015). The move towards adopting professional learning communities as an alternative requires knowledge of what teachers can learn, and what supports learning in such communities.

Research on professional learning communities has focused mainly on understanding what and how practising teachers learn in such communities and the extent to which such learning might improve teachers' practices (Brodie, 2014; Brodie & Borko, 2016b; Chauraya & Brodie, 2017; Goldsmith, Doerr, & Lewis, 2014; Horn & Little, 2010; Koellner-Clark & Borko, 2004). Most studies highlight the significance of inquiry-focused conversations for learning in a professional learning community, but very few have analysed in detail the learning opportunities created in such conversations. This article addresses this gap by presenting analyses of the conversations of a professional learning community and how they provide learning opportunities. The questions addressed in the article are: What teacher learning opportunities are created in the conversations of a professional learning community? What are the characteristics of the practice of a professional learning community that support learning opportunities?

Teacher learning in professional learning communities

In this article we regard a professional learning community as 'a group of teachers sharing and critically interrogating their practice in an on-going, reflective, collaborative, inclusive,

learning-oriented, growth-promoting way' (Stoll & Louis, 2007, p. 2). In our study the professional learning community consisted of five mathematics teachers in one high school and the first author as facilitator. The community engaged in a number of developmental learning activities (see Brodie & Shalem, 2011, for more details), which involved conversations about their understandings of learner errors in mathematics. Our analyses of these conversations in this article sought to understand the learning opportunities that were created.

The theoretical framework that guided the study was situated learning theory (Lave, 1991; Lave & Wenger, 1991; Wenger, 1998), which regards learning as situated in human activities, inseparable from the environments in which knowledge is used, and occurring through social processes that involve the negotiation of meanings with others (Buysse, Sparkman, & Wesley, 2003). According to the theory learning is situated in communities of practice, including professional learning communities, which we regard as a special kind of community of practice (Brodie & Borko, 2016a) that occurs in the practices of such communities (Wenger, 1998). For Wenger the practice of a community of practice has three dimensions or characteristics, which are a source of coherence for the community: joint enterprise, mutual engagement and a shared repertoire. The joint enterprise is the shared basis for action with colleagues, or the 'negotiated response to their situation' (Wenger, 1998, p. 77) which the community strives to resolve or understand. In a professional learning community the joint enterprise is akin to a learning focus, which is an issue of concern, or a problem of practice that the teachers inquire into so as to develop new understandings that deepen their professional knowledge and practice (Katz & Earl, 2010). In this study the learning focus for the professional learning community was understanding learner errors in mathematics. The second dimension of practice is mutual engagement which refers to active collaboration involving such practices as sharing understanding, joint reflection, collective development of new meanings and joint decision-making (Katz & Earl, 2010). According to Wenger (1998) mutual engagement fosters coherence in a community of practice and is a source of new meanings and understandings for the community, also called the shared repertoire. The shared repertoire, as the third dimension of the practice of a community of practice, 'includes routines, words, tools, ways of doing things, stories, gestures, symbols, genres, actions or concepts that the community has produced or adopted in the course of its existence, and which become part of its practice' (Wenger, 1998, p. 83). In a professional learning community the shared repertoire consists of the meanings that are developed and ways of dealing with, or resolving, situations that are jointly negotiated and adopted as part of the community's practice.

The three dimensions of practice are presented in Figure 1, which shows how having a shared learning focus and engaging in collective action supports mutual engagement activities which in turn lead to the development of new meanings and ways of addressing problems of practice. The three dimensions shape each other, hence the double arrows.

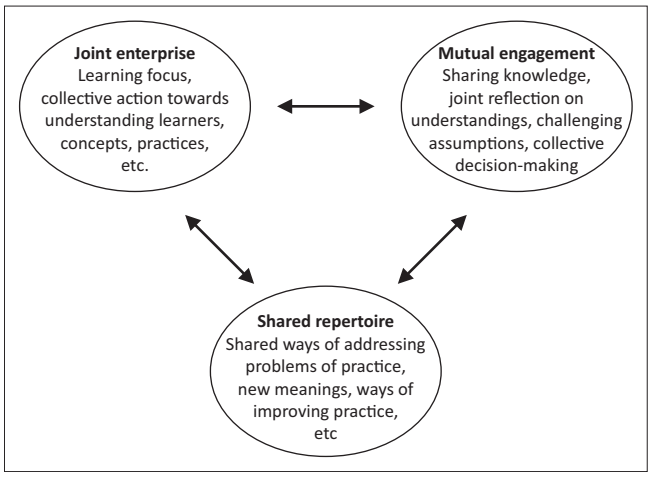

FIGURE 1: Dimensions of practice in a community of practice.

In this article we adopted these three characteristics of practice as one of the two frameworks or analytical tools that we used in analysing the conversations of the professional learning community.

Our goal in using this framework was to understand the teachers' participation in the professional learning community as a practice that supported learning opportunities in the teachers' conversations, which were the data that we analysed in this article.

Teacher knowledge in a professional learning community

Another aspect of teacher learning in a professional learning community that was of interest to us was what teacher knowledge could have been learned in the conversations of the community. To analyse this knowledge we used *pedagogical content knowledge* and *content knowledge*, which are seen as part of teachers' *mathematical knowledge for teaching* (Ball, Hill, & Bass, 2005; Ball, Thames, & Phelps, 2008; Hill et al., 2008). Mathematical knowledge for teaching is defined as the knowledge that mathematics teachers need to know in order to teach the subject well, and includes teachers' knowledge of: the content of mathematics, the different representations of the mathematics so that it is accessible to learners, learners' mathematical thinking, and how to make in-the-moment decisions that support learners' learning. So our second analytical tool consisted of pedagogical content knowledge and content knowledge as part of teachers' mathematical knowledge for teaching. Such knowledge is part of teachers' pedagogical content knowledge and, as we will show, is related to their content knowledge.

The two analytical frameworks presented above have a close relationship with each other that enabled us to analyse both the teachers' participation in the conversations and what they learned in those conversations. The joint enterprise, mutual engagement and shared repertoire framework enabled us to analyse the teachers' participation as a practice of the community. The mathematical knowledge

for teaching framework enabled us to analyse the knowledge implicated in the conversations. For us the two frameworks are related in that in each conversation episode the mathematical knowledge was part of the learning focus or joint enterprise, and through active participation in the conversation, there were opportunities for deepening the teachers' knowledge and developing new meanings about the teachers' mathematical knowledge for teaching.

Inquiry in professional learning communities is usually supported by a facilitator or 'critical friend', who participates in the professional learning community and supports learning by asking for critical reflection on issues, probing for justification, challenging assumptions, pushing for deeper thinking and interpretations, reminding participants of what has been achieved, and guiding the direction of the ongoing conversations (Brodie, 2016; Katz, Earl, & Jaafar, 2009). A facilitator thus plays a fundamental role in supporting teacher learning and in this article we also analysed the facilitator's role in the learning opportunities created in the professional learning community.

Research findings on teacher learning in professional learning communities

Research on teacher learning in professional learning communities shows that learning can occur in some important dimensions of teachers' knowledge and practice. In some communities teachers learned to analyse mathematical problems in greater depth (e.g. Borko, Jacobs, Eiteljorg, & Pittman, 2006). Learning was also observed in teacher reflection on classroom practices in ways that could result in changes in practice (Horn, 2010). Some studies have shown how teachers learned about their learners' thinking in making errors and developed better understandings of learners' thinking (e.g. Brodie, 2014; Kazemi & Franke, 2004). Other studies have shown changes in teachers' knowledge of the curriculum (Brodie, Shalem, Sapire, & Manson, 2010), and the teaching practices (Tam, 2015). In all the studies, teacher learning was attributed to participation and collaboration in teacher learning communities.

In South Africa findings from a teacher professional development project have shown that conversations in professional learning communities provided opportunities for developing both content knowledge and pedagogical content knowledge, that content knowledge and pedagogical content knowledge conversations often triggered each other (Marchant & Brodie, 2016), and that the different project activities supported talk on different foci: learners, mathematics and practice (Chimhande & Brodie, 2016). Teacher participation in the professional learning communities supported the deepening of teachers' mathematical knowledge (Brodie, 2014; Liebenberg, 2016), shifts in teachers' talk about learner errors (Brodie, 2014), shifts in teachers' practices (Chauraya & Brodie, 2017; Molefe, 2016) and shifts in teachers' identities (Chauraya, 2016). Other studies involving teacher learning communities in South Africa have shown that teachers developed confidence that was central to their learning (Graven, 2004).

The findings cited above highlight how teacher communities can support teacher professional learning in terms of understanding and addressing problems of practice, understanding learners' thinking, developing new teaching practices, and deepening knowledge of mathematics and the curriculum. The studies also highlight the significance of collaboration, community and inquiry to learning in such communities. Our article contributes to this growing knowledge base by presenting findings of analyses of learning opportunities created in conversations about learner errors.

The study methodology

The research reported in this study involved a school-based professional learning community of mathematics teachers, which was a pilot to phase three of the Data-Informed Practice Improvement Project. The project establishes and supports professional learning communities of mathematics teachers in high schools in the Johannesburg area. The underlying goal of the project is to develop teachers' pedagogical content knowledge in order to support changes in both their teaching practice and their content knowledge. To address this goal, the project has developed a set of activities that supports teachers' learning about how to engage with learner errors in ways that provide access to mathematics for both learners and teachers (Brodie, 2013).

In the study, five mathematics teachers in one township high school, together with the first author of this article as facilitator, worked as a school-based professional learning community on the professional learning activities. The professional learning community met once a week for two hours after school, during term time. The professional learning activities were: analysing the errors that learners made on tests with a view to understanding the learners' reasoning behind the errors, interviewing selected learners on errors they had made to understand their reasoning in more depth, identifying learners' learning needs in the form of critical mathematical concepts, reflecting on teachers' own understanding of the mathematics concepts, designing and teaching mathematics lessons in those concepts, and reflecting on videotapes of the lessons that were taught with the aim of understanding how the teachers engaged with learner errors in their teaching. A critical concept is a mathematical concept that underlies a number of key areas of the curriculum, and learner difficulties with the concept can contribute to patterns of errors linked to the concept. The critical concept in this article is ratio, and it was identified as critical for learners through analysis of the learner errors that were observed on test items (see Chauraya, 2013).

The community analysed learner errors on a number of items on ratio, and we present one here that we selected as representative of the analyses. The data for this article

consisted of episodes of conversations on learner errors on the topic of ratio, and learner interviews that were conducted by one of the teachers on this item. Learners were selected for interviews according to the errors they made, and which errors the community identified as requiring more detailed information about the possible reasoning behind the errors. Each teacher interviewed between two and four learners. In this article we focus on one teacher's interview with one learner. These data were collected through audio recordings that were later transcribed for analysis.

Data analysis

Data analysis involved three stages. The first stage involved identifying the conversation episodes for analysis from the lengthy conversations of the community. The second stage involved summarising the selected episodes for presentation and analysis. The last stage involved the actual analysis of each episode using the analytical frameworks presented earlier in the article.

In identifying the conversation episodes for analysis in this article we adopted the critical incidents approach which is a qualitative data analysis technique mainly used in analysing classroom events in order to understand teacher actions and reflections in instructional situations (Angelides, 2001; Yang & Ricks, 2012). A critical incident in school situations is regarded as a commonplace event that occurs in the everyday life of a classroom but is justified as critical on the basis of its significance to the goals of the activity and the researcher's interpretation of the event (Angelides, 2001). The criticality of an incident depends on the 'justification, the significance, and the meaning given' by the researcher (Angelides, 2001, p. 431). Thus a researcher looks at the underlying trends, motives and structures in the incident and ascertains whether it is critical or not depending on what they are interested in before analysing the teaching or learning effect of the episode (Yang & Ricks, 2012).

In adopting the critical incidents approach we regarded a critical episode as a conversation focused on one of the following themes: making sense of learners' reasoning through analysis of learner interviews, understanding learners' errors and identifying the critical concept for learners, and the teachers' own understanding of the critical concept. These themes were deliberately chosen in line with the goals of the Data-Informed Practice Improvement Project which included developing teachers' knowledge of learner errors and how to work with errors in their teaching.

In presenting and summarising each selected episode we identified only those conversation turns that contributed to the theme of the episode. A turn is an utterance or comment by a participant in a conversation. Turns that were not relevant to the theme of the episode were disregarded. Having identified the relevant turns in each episode, we then categorised these turns according to what was being said and the purpose that each turn served. For example a question by the facilitator was categorised as 'facilitator question', and then according to whether the question served to challenge the teachers' knowledge or probe for deeper meaning. Thus our presentation summarises the selected conversation turns in each episode in tabular form, showing the nature and frequency of particular categories of conversation turns, including examples of such turns.

As indicated earlier, in the actual analysis we used two frameworks or analytical tools. The dimensions of practice in the community as depicted in Figure 1 enabled us to analyse: what formed the focus of each conversation episode (the joint enterprise), how the teachers engaged in the conversation (mutual engagement), and the nature of the developing new meanings or practices (the shared repertoire). The mathematical knowledge for teaching framework enabled us to identify the nature of mathematical knowledge in each conversation episode and any new meanings that could have been developed about that knowledge.

Ethical consideration

The relevant university research ethics committee cleared the project from which the data and findings reported in this article were drawn. The ethics clearance number was 2009ECE171. The Gauteng Department of Education approved the project through a letter detailing the conditions for the conduct of the project in schools in the province. To maintain anonymity all the names of participants in this article are pseudonyms and the name of the school is not mentioned anywhere in the article.

Findings

The episodes that we analyse below were drawn from the conversations about learners' errors on the following ratio test item:

Divide R800 in the ratio 5:3.

The item was in a test given to Grade 10 and Grade 11 learners. The most common observed errors on this test item were as follows:

$\frac{800}{5}$ and $\frac{800}{3}$, $\frac{800}{5:3}$ and $\frac{800}{5}:3$.

Individual inquiry into learner reasoning: Learner interviews

As part of the analyses of learners' errors, each teacher interviewed selected learners on specific errors that they had made. Learner interviews were intended to support the teachers to understand learner errors more deeply, in particular what was both valid and not valid in the learners' reasoning. The interviews were also the initial step for the teachers in trying to make sense of the learners' errors and were central to conversations about learner errors.

Table 1 shows an extract of one teacher's interview with a learner on the error: $\frac{800}{5}$ and $\frac{800}{3}$. All the teachers' and learners' names are pseudonyms. Janeth is the teacher in these extracts.

Interviewing learners about their reasoning was not part of teachers' practice prior to the professional learning community. In preparation for learner interviews, the professional learning community had discussed the need to ask and follow up on questions that would elicit the substantive learner reasoning behind the errors. The teachers role-played interviews and learned how to distinguish between questions that tried to guide the learners to the correct answer and questions which served to access the learners' reasoning. In the episode in Table 1 the teacher started off with a general question 'Can you explain how you solved this problem?' After the learner explained his strategy, the teacher probed further for the learner's reasoning, which the learner struggled to provide – he kept talking about his strategy. Even the teacher's question 'Why did you do that?' produced the answer 'Because I wanted to get the final answer'. Our data is replete with such examples, with learners struggling to make their reasoning explicit and teachers struggling to support them to do this. The teacher was responsive to the learners' difficulties, and shifted her question to 'But this idea of dividing, where did divide come from?' This question supported the learner to say that he was guided by the word 'divide' in the problem, which shows the teacher where some of the difficulty lies.

This interview extract shows how the learner interviews were learning opportunities for the teachers to learn how to elicit, in more detail, learners' thinking behind observed errors. Through the interviews the teachers also had opportunities to begin to understand why learners make errors in mathematics, and how the errors are reasonable for learners (Brodie, 2014). This is an important aspect of teachers' pedagogical content knowledge, which they can work with in their teaching as they respond to learners' errors and learners' learning needs. However in the interviews the teachers often missed opportunities for getting more insights into the learners' understandings of some mathematics concepts. In the episode above the teacher did not ask the learner to explain his understanding of the ratio '5:3', thereby missing the opportunity to get the learner's understanding of the concept.

Collective inquiry into learners' reasoning: Community conversations

In the professional learning community initial conversations about learner errors focused on understanding the learner errors, giving the possible reasons for the errors, and identifying the critical concepts for the learners, also known as the learners' learning needs. The learner interviews served to provide evidence for the learners' thinking in making errors. Understanding learners' reasoning in errors formed the learning focus or joint enterprise for the community in these conversations. The conversation presented in Table 2 shows how the teachers mutually engaged in trying to make sense of the learners' thinking in making the observed errors.

In our analysis below we use an ordered pair notation, for example (T5, Sibanda); the first value Ti refers to conversation turn number and the second value refers to the pseudo names of the participants.

The episode shows certain patterns of interactions in the community conversations which were also evident in other and similar conversations. The episode was characterised by the facilitator's questions that served to initiate and probe for deeper thinking, the teachers' interpretations, explanations and justification of learner errors, and elaborations by the facilitator as a way of summarising the meanings being developed. These patterns of interaction characterised the mutual engagement by the teachers in the conversations. The conversation drew from the learner interview presented earlier on, which illustrates how learner interviews were useful data that supported the joint enterprise. The larger part of the episode consisted of turns in which the teachers interpreted the error and proposed reasons for the error. Interpreting the error involved identifying what the learners

TABLE 1: Extract of a learner interview.

Turn number	Speaker	Utterance
1	Janeth [*teacher*]	So Melusi, the question was divide eight hundred in the ratio five is to three. Can you explain to us how you solved this problem?
2	Melusi [*Learner*]	I took eight hundred and I divided it by five and it gave me one sixty, and I took it again eight hundred and I divided it by three and it gave me two sixty, no two thousand six hundred, no two hundred and sixty, eh no maam this answer is wrong [*referring to his computational error in dividing 800 by 3*].
3	Janeth	Why do you say it is wrong?
4	Melusi	Maam I made a mistake.
5	Janeth	But why did you divide eight hundred by five and then eight hundred by three?
6	Melusi	I take this number and I divide it into eight hundred and I take this number and I divide it into eight hundred.
7	Janeth	My question is why did you do that?
8	Melusi	Because I want to get the final answer.
9	Janeth	But this idea of dividing, where did divide come from?
10	Melusi	I don't remember why I divided by this number; no it's because of this word divide, that is why I divided by these numbers.
11	Janeth	Because of the word divide?
12	Melusi	Yes.
13	Janeth	So you think that when it says divide eight hundred in the ratio five is to three it means divide eight hundred by five and three?
14	Melusi	Yes.

TABLE 2: Understanding a learner error.

Category of utterance	Purpose	Example	Number of turns
Facilitator questions	Initiation	What are the errors that we observed? (T1, Sibanda)	1
	Probing	Why did the learners make this error? (T5, Sibanda)	2
Teacher reports	Informing others	For division, they said they saw the word divide (T6, Janeth)	3
Teacher explanations	Interpreting an error	They divided eight hundred by those numbers (T14, Mandla) They threw away the word ratio (T14, Bongiwe) (T20, Bongiwe)	2
	Explaining the error	And according to the language he is right (T11, Bongiwe) One learner could not explain what it means to divide in the ratio (T16, Janeth)	5
	Justifying an error	Maybe our problem is these learners forget too much (T20, Bongiwe)	2
Facilitator explanations	Elaborating teacher explanation	Ja, they threw away the word ratio … When we interviewed some of them they didn't know what it means to say five is to three (T15, Sibanda) They don't know what ratio is (T28, Sibanda)	9

did in making the error. Explaining the error involved suggesting the possible reasoning behind the error, and in this case the explanation was that the learners took 'divide in the ratio …' to mean 'divide by …'. One teacher's comment that this was correct according to the language suggests that she was beginning to see the reasonableness of the error from a learner's perspective, which is an important step for teachers in learning how to view errors as integral to learning mathematics. However, the same teacher's suggestion that the error could be justified in terms of learners forgetting what they had been taught was an example of blaming learners for errors, something that shows limited understanding of the nature of errors in mathematics learning.

This episode illustrates how conversations about learner errors created opportunities for learning how to interpret learner errors and explain the thinking behind the errors, as well as identifying learners' learning needs from their errors – all examples of opportunities for developing pedagogical content knowledge mutual engagement through practices such as facilitator questions and teachers' responses and explanations which supported opportunities for learning about learners' thinking in making errors. The teachers' explanations of the error served as examples of developing new thinking or understanding about learner errors, which could be part of the teachers' developing shared repertoire.

The facilitator's central role in this episode was that of initiating and probing through questions, as well as clarifying and building on the teachers' contributions, all of which supported mutual engagement in the conversation. In the next section we show how the conversations shifted to the teachers' subject content knowledge of ratio.

Collective inquiry: The teachers' understanding of ratio

One of the errors was attributed to learners not understanding what it meant to divide a quantity in a given ratio. In the following episodes we analyse how the facilitator shifted the conversation from the identification of learners' learning needs to teachers' learning needs. Two critical episodes in the conversation focused on the teachers' understanding of ratio as a concept and the conceptual meaning of the steps in the algorithm for dividing quantities in given ratios. Both episodes were on the teachers' subject content knowledge of ratio. Table 3 shows a summary of the critical turns in the conversation and how we categorised the turns according to how they contributed to the theme of the episode.

The facilitator's questions served to support the teachers to share their individual understandings of ratio. The teachers' responses showed an algorithmic understanding of ratio, in which they described ratio in terms of the steps in the algorithm for dividing a quantity in a given ratio. The ensuing funneling questions by the facilitator were a way of getting the teachers to the concept, upon realising that the teachers were not likely to give the conceptual meaning. In the last turns of the episode the facilitator elaborated the concept further as a way of supporting the teachers' understandings.

Although there was no evidence that the teachers learned the conceptual meaning of ratio, the episode was an opportunity for the teachers to develop a new shared understanding of the concept of ratio, thus deepening their subject content knowledge. The data above show that in an effort to shift the teachers' algorithmic understanding to an understanding of the concept of ratio, which includes the algorithm, the facilitator deliberately used funneling questions. We acknowledge the limitations of funneling questions, and in this case there was no evidence that this move by the facilitator resulted in deeper learning of the concept by the teachers.

The conversation episode summarised in Table 4 focuses on the teachers' understandings of the conceptual meaning of the steps in the algorithm for dividing a quantity in a given ratio. Table 4 presents a summary of the conversation and our categorisation of the critical turns.

The facilitator's challenging questions and the teachers' expressions of their own understandings were the main forms of mutual engagement in this episode. The teachers' responses to the initial question by the facilitator showed knowledge of the steps in the algorithm, but they could not explain the conceptual meaning of the algorithm as a whole. The teachers' difficulties in explaining the algorithm could be attributed to how they normally teach the algorithm. Mathematics teachers in South Africa often teach the algorithm without worrying about its conceptual meaning, and knowledge of the steps in the algorithm is considered to be sufficient and relevant for the curriculum and learners' examination needs (Mogari, 2014). In not regularly thinking

TABLE 3: Teachers' understanding of the ratio concept.

Category of utterance	Purpose	Example	Number of turns
Facilitator questions	Initiation	What is it that learners understand about ratio? What is ratio? (T1, Sibanda)	1
	Re-stating main question	It boils down to our understanding of ratio, what is ratio? (T19, Sibanda)	4
	Funneling	What is the ratio of men to women in this school? (T42, Sibanda) What are we doing? (T44, Sibanda)	2
Teacher responses	Showing teachers' initial understandings of ratio	Ratio is dividing in parts (T2, Jeffrey) Ratio is total parts of a whole (T13, Jeffrey) We are sharing, it's a share (T28, Janeth) It's a way of sharing (T33, Bongiwe) We are sharing, but not in equal parts (T35, Bongiwe) You are distributing (T37, Mandla)	16
	Correct meaning of ratio	We are comparing (T45, Bongiwe)	1
Facilitator elaboration	Affirming and explaining further the concept of ratio	Ratio is a way of comparing quantities (T46, Sibanda) When they say the ratio of teachers to learners is one is to thirty-five, all they are saying is for every teacher there is thirty-five learners (T51, Sibanda)	4

TABLE 4: Teachers' understanding of the ratio algorithm.

Category	Purpose	Example	Number of turns
Facilitator questions	Challenging knowledge	If you are dividing a quantity in a given ratio, what basically are you doing? Let us use the example, divide eight hundred rands in the ratio five is to three (T1, Sibanda) Can we have a conceptual understanding, what is happening there? What does it mean? (T5, Sibanda)	2
	Re-stating and re-phrasing the question	So the process of dividing eight hundred in the ratio five is to three, physically what can we say is happening? If we say do it physically how would you do it? (T11, Sibanda)	13
Teacher responses	Expressing own understanding	I am going to put the total number of people that I have to share the eight hundred (T14, Bongiwe) The total is the meaning of a whole, the whole consists of eight parts, five and three (T16, Jeffrey) I will say this one you are five, you are three, and I give you your five hundred and you get your three hundred (T32, Mandla)	7
Teacher questions	Seeking clarification	You are asking for the process? This is what is not clear (T22, Mandla) There is a question, how did you get hundred times? (T90, Jeffrey)	10
Facilitator explanation	Explaining meaning of algorithm	What it means is that for every three that I give this one I must give five to the other one. The next time I give this one three, I must give five to the other, and I do that until the whole amount is exhausted. That's sharing in the ratio five is to three (T58, Sibanda) The question now becomes how many times must I do that, and in the end how much does each get? (T60, Sibanda) The ratio five is to three means every five I give her I must give three to him. So how much am I giving out each time? (T66, Sibanda) If I have eight hundred to give out how many times should I do that? (T83, Sibanda) In other words that is the conceptual meaning of dividing a quantity in a given ratio (T122, Sibanda)	27
Teacher responses	Attention to facilitator's explanation	Every time you give out eight (T67, Bongiwe) Hundred times (T86, Tintswalo)	24
Teacher explanation	Explaining to others	From eight hundred we have to take out eight all the times, so eight hundred divide by eight what do you get? How many eights are there in hundred? (T94, Tintswalo)	1
Teacher responses	Indications of understanding	Oh, this five plus three (T99, Mandla) Oh okay hundred times (T101, Mandla) And hundred divide by eight (T103, Janeth)	7
Teacher remarks	Expressing linkage to teaching	I can start tomorrow, delivering to the learners (T135, Mandla) But this is a nice way of teaching (T138, Bongiwe)	4

about the concepts behind the algorithm, the teachers come to see the algorithm as the concept. Brodie and Sanni (2014) give similar examples in which Nigerian teachers' pedagogical content knowledge and knowledge of the curriculum constrained conceptual reflection on their content knowledge.

The episode culminated with the facilitator sharing his own knowledge of the algorithm, a move that was necessitated by his realisation that the teachers could not explain the conceptual meaning of the algorithm as a whole. This move by the facilitator was necessary for purposes of building professional knowledge, an important aspect of learning in a professional learning community and a practice that was also evident in other conversations that we analysed. After the facilitator's explanation, remarks by Mandla and Bongiwe, while not necessarily showing their own conceptual understanding of ratio, indicate that they saw the conversation as developing their knowledge of teaching the ratio algorithm differently.

In terms of their mathematical knowledge for teaching, the episode was an opportunity for the teachers to deepen their understanding of the algorithm, and hence their pedagogical content knowledge. We acknowledge that there was no evidence to indicate that they had learned this knowledge, but two teachers' expressions of their willingness to work with this knowledge in their teaching could be indications of new and shared understandings that could influence their teaching.

Discussion and conclusions

In this article we have analysed the teachers' participation as a practice of the community and the nature of the mathematical knowledge that they could have learned in the conversations of the community. These analyses enabled us to respond to both the research questions that we set in the article.

The episodes presented above show how a focus on understanding learners' errors in the topic ratio developed into conversations about the teachers' mathematical knowledge for teaching (Ball et al., 2008), and these foci constituted the joint enterprise for the community. Mutual engagement was evident through the teachers' responses to

the facilitator's questions that served to challenge and probe the teachers' understandings as a way of pushing for the development of new understandings. In the episodes we presented there were opportunities for the teachers to: understand learners' thinking behind errors, interpret, explain and justify learner errors, thus becoming more aware of the origins of learner errors, and identify learners' learning needs, from a variety of evidence. These opportunities indicate the potential for professional learning communities to develop new and shared understandings, or a shared repertoire for the teachers. Our findings show that having a clearly defined, shared and understood joint enterprise, and mutually engaging with others in seeking to develop new understandings, are significant to the creation of learning opportunities in professional learning communities. Such learning opportunities have the potential to develop a shared repertoire that can contribute to teachers' knowledge and practice. The three dimensions of the practice of a community of practice (Wenger, 1998) presented in Figure 1 enabled us to analyse and characterise the patterns of participation in these conversations. For us these forms of participation typified the community conversations as a developing practice in which learning opportunities are created and supported.

Our analyses of the conversation episodes presented in this article show that the conversations were opportunities for developing the teachers' knowledge of learners' reasoning in making errors and the reasonableness of learners' errors, and their own conceptual understanding of ratio and the algorithm for dividing quantities in given ratios. In the conversations the teachers showed that they had an algorithmic understanding of ratio as evidenced by their responses to the facilitator's questions, which is consistent with how the topic is normally taught in mathematics classrooms where teachers emphasise knowledge necessary for examinations, which in mathematics is mainly procedural fluency (Mogari, 2014). Teachers have a tendency to limit their understandings to what they are used to teaching at the expense of conceptual meanings of topics (Brodie & Sanni, 2014). Although there is no evidence to suggest that the teachers did learn about the conceptual meanings of ratio and the ratio algorithm, the conversations were opportunities for the teachers to deepen their own knowledge of the topic through negotiation of meanings with others in the community (Buysse et al., 2003). We therefore maintain our argument that an opportunity for learning the conceptual meaning of ratio was created and if this supports teachers to teach in a new way, it may support gradual development of their conceptual understandings. In short, the episodes were opportunities for deepening the teachers' knowledge of the subject, knowledge of their learners, and ways of teaching the topic ratio. These are important aspects of teachers' mathematical knowledge for teaching which can support productive teaching of the subject, and this indicates the potential for professional learning communities to support new understandings for teachers in knowledge that is important for their practice.

The facilitator was key to the learning opportunities provided in the community. The conversations show instances where the facilitator-initiated conversations challenged teachers for deeper thinking or understanding, guided the focus of each conversation, and provided knowledge when necessary. For example, in the conversations, the link between learners' learning needs on the topic of ratio and the teachers' own knowledge was raised by the facilitator. The teachers were not able to work from their learners' learning needs to identify and reflect on their own learning needs without a strong intervention from the facilitator. These interventions by the facilitator have been found to be fundamental for the effective functioning of professional learning communities and underscore the critical role of facilitators in such communities (Katz, Earl, Jaafar, Elgie, & Foster, 2008).

Our arguments resonate with those of other researchers who have highlighted the power of professional learning communities to influence teacher learning. We have shown how conversations in professional learning communities can create learning opportunities for the participating teachers in significant aspects of their mathematical knowledge for teaching. The critical role of a facilitator in professional learning communities was shown by the way in which he initiated the teachers' reflection on their own understandings, thereby creating opportunities for them to develop new meanings. We have also illustrated how learning in professional learning communities is supported by a joint enterprise, mutual engagement, and the development of a shared repertoire (Wenger, 1998). We argue that this article contributes to knowledge of how learning opportunities can be created in professional learning communities, a teacher professional development approach which is gaining popularity, especially in the South African context where developments are being made to introduce such communities in schools.

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Competing interests

We declare that we have no financial or personal relationships that may have inappropriately influenced us in writing this article.

Authors' contributions

The article arose from a four-year research project in which K.B. was the principal investigator and M.C. was a doctoral student, with K.B. as his supervisor. This article was written jointly by both authors.

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